Electron Cloud Instability

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Clouds and Clocks

- “My clouds are intended to represent physical systems which are highly irregular, disorderly, and more or less unpredictable. I shall assume that we have before us a schema or arrangement in which a very disturbed or disorderly cloud is placed on the left. On the other extreme of our arrangement, on its right, we may place a very reliable pendulum clock, a precision clock, intended to represent physical systems which are regular, orderly, and highly predictable in their behavior… There are lots of things, natural processes and natural phenomena, which we may place between these two extremes - the clouds on the left, and the clocks on the right.”

  Karl Popper, “Of Clouds and Clocks, an approach to the problem of rationality and the freedom of man”, to a memory of Arthur Compton.

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How to describe clouds?

- Electron clouds are irregular, poorly reproducible, very complicated phenomena. They are popperian clouds indeed. That is why it does not seem reasonable to make efforts for detailed depicting of their forms – the cloud changes faster than the artist is able to image its contour.

- We may still hope to catch roughly the main parameters of these objects, being able at least very approximately orient ourselves between them.

- The more complicated and irregular is the object, the simpler is its reasonable mathematics. Let's try to stick with simple estimations, and do not loose main factors.

Main Factors

- E-cloud influences incoherent and coherent oscillations of beam particles in various aspects.

  - It works as a static lens, shifting up all coherent and incoherent tunes.

  - It gives a significant tune spread. With the size of the e-cloud similar to the proton beam size, the nonlinear tune spread is comparable to the tune shift. The EC tune spread as well as SC are important for the Landau damping (LD) .

  - As a reactive medium, e-cloud works as a sort of low-Q impedance.

- Thus, e-cloud introduces both the impedance and LD / decoherence.

  Who of them is going to win?

RR Parameters (PIP-2)

Recycler	Requirement	
Bunch population, N	$8.2 \cdot 10^{10}$	
Number of bunches	$81 \cdot 6 \cdot 2 = 972$	
Transverse emittance, norm. rms, $\varepsilon_{n\perp}$	2.5	mm mrad
Longitudinal emittance, rms, $\varepsilon_{\parallel} = \sigma_\tau \sigma_E$	3.6	meV s
Maximal RF Voltage , V	0.125	MV
Transition gamma γ_t	21.6	

$$Q_s = 0.0027; \quad \Delta Q_{sc} = 0.1; \quad \delta p_{rms} / p = 3.3 \cdot 10^{-4}; \quad \sigma_s = 55\text{cm}.$$

Static focusing

- We assume the relevant e-cloud transverse size equal to the proton size. The incoherent tune shift follows:

$$\delta\nu_{pe} \simeq \frac{\pi n_e r_p R_0^2}{\gamma\nu_b} = \pi n_e r_p R_0 \bar{\beta} / \gamma$$

- The rms spread of the tune shifts is assumed comparable to its average value.

- Tunes for all the beam coherent modes are going up as well due to this, by similar values.

Wake function

- Following [Burov & Dikansky,1997], e-cloud wake can be modeled as a low-Q resonator:

$$W(\tau)\;;\;W_0 \sin(\omega_e \tau)\exp\left(\omega_e \tau / 2Q\right);$$

$$W_0 = \frac{4\pi n_e r_e c L}{a^2 \omega_e}, \quad Q \sim 2-3, \;\; \tau < 0$$

equivalent to a shunt impedance $\frac{R_s}{Q}\;;\;Z_0 \frac{n_e r_e c L}{a^2 \omega_e};\;Z_0 = \frac{4\pi}{c} = 377\,\mathrm{Ohm}$

Here n_e is the average e-cloud density inside the proton beam size of the radius a , L is the length of the e-cloud affected part of the machine,

$$\omega_e = (c/a)\sqrt{N_b r_e / \sqrt{2\pi}\sigma_z}$$

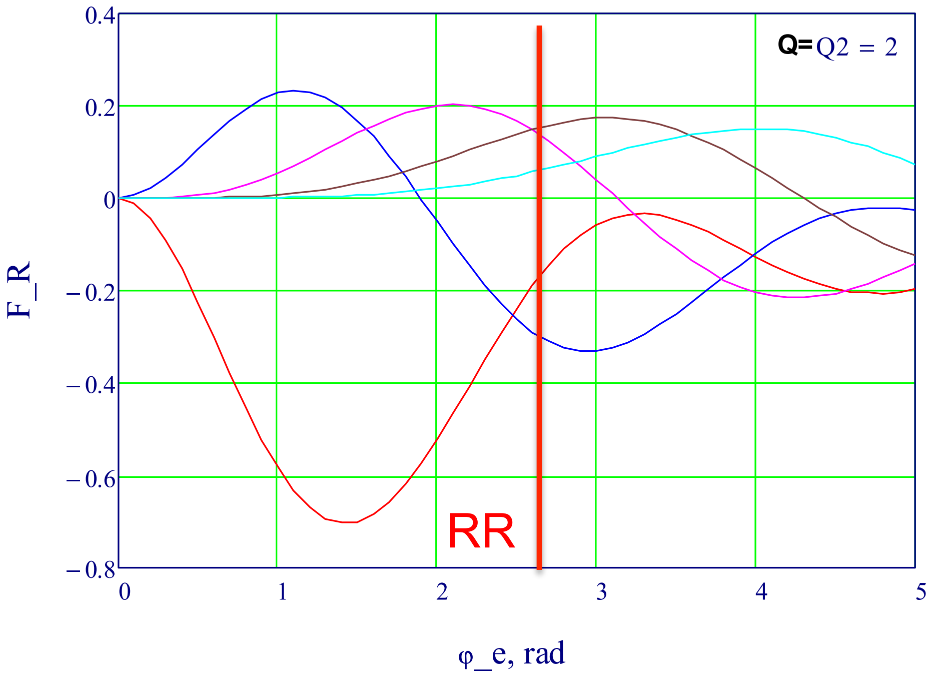
is the frequency of electron oscillations in the space charge field of the bunch with rms length σ_z .

Weak Head-Tail (WHT)

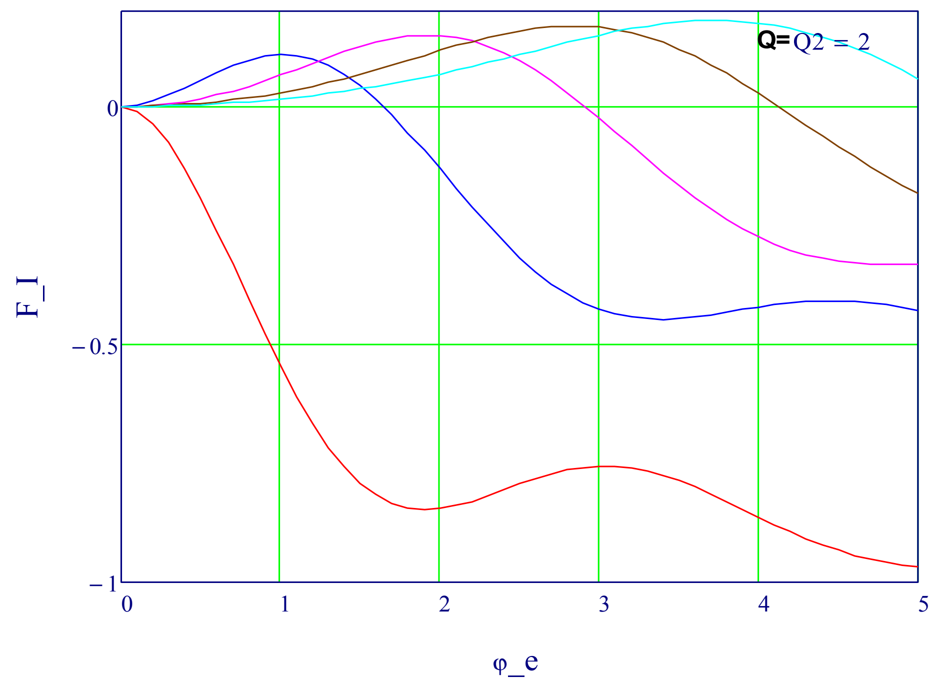
- Application this wake function to the WHT tune shift and growth rate (A. Chao, Eq. 6.213, air-bag) results in (HT phase $\chi \le 1$) :

$$\mathrm{Im}[\Delta\nu_m] = \chi\delta\nu_{pe}F_R(m,\phi_e); \quad \mathrm{Re}[\Delta\nu_m] = \delta\nu_{pe}F_I(m,\phi_e); \quad \phi_e = \omega_e\sigma_z / c$$

Growth rates factors vs BB wake phase advance



Mode tune shifts factors vs BB wake phase advance



$$F_R(m,Q,\phi) = 3\int_0^\infty \frac{J_m(x\phi)J'_m(x\phi)}{1+Q^2(x-1/x)^2}\frac{dx}{x}; \quad F_I(m,Q,\phi) = \frac{3}{2}\int_0^\infty \frac{Q(x-1/x)J_m^2(x\phi)}{1+Q^2(x-1/x)^2}\frac{dx}{x}$$

WHT (2)

- These rates and phase advances have to be compared with the LD.
- Without space charge, LD is driven by the e-cloud rms tune spread

$$\delta_2 \nu_{pe} \simeq (0.3-0.5)\delta\nu_{pe}$$

- Thus, it can be concluded:
  - HT mode 0 can be unstable: its tune shift can exceed the spread, and its rate can be high. But for the proper sign/value of the chromaticity it is damped. Also it can be damped by a damper.
  - Higher HT modes are damped by LD – at least at $\chi < 1$, since $\delta_2 \nu_{pe} > \mathrm{Im}[\Delta\nu_m]$
  - Thus, all the HT modes can be damped (most likely are L-damped at any $\chi > 0$).
  - If there is SC so high that $\delta\nu_{sc} > \delta\nu_{pe}$, it kills LD, and WHT becomes possible.

Fast Instability in the RR

- Instability in the RR is NOT weak head-tail. It is very fast: 20 revolutions of the growth time means it is 30 times faster than the synchrotron oscillations! Thus, the WHT growth rates above are not applicable.

- It is beam breakup (BBU) type transverse instability.

Beam Breakup (BBU)

- When the electron phase advance is not small, $\phi_e \geq 1$, the BBU growth rate can be estimated as

$$\mathrm{Im}[\Delta\nu_{\mathrm{bbu}}] \simeq \delta\nu_{pe},$$

comparable to the estimated nonlinearity.

$$\delta\nu_{pe} = 0.01 \Rightarrow n_e = 1 \cdot 10^6 \mathrm{cm}^{-3}$$

- Whether BBU is winning over e-cloud nonlinearity is an open question.
- BBU can be stabilized by high chromaticity if

$$\xi \delta p / p \geq \delta\nu_{pe}.$$

- With $\delta p / p = 0.0003$, it requires $\xi \geq 30.$

Transverse HOMs in RR?

- Can this RR instability be driven by HOM?
- The growth rate

$$\tau^{-1} = \frac{N_{\text{total}} r_0}{R_0 \gamma} \frac{R_s \bar{\beta}}{Z_0 T_0}$$

for 20 turns of the growth time, 3E12 protons, yields the shunt impedance

$$R_s \bar{\beta} = 2\text{G}\Omega.$$

This requires the HOM Q value~10^6, or 1.5-2 orders of magnitude higher the maximal expected values.

Thus, HOM cannot be responsible for the RR instability.

Conclusions

- The fast transverse instability in the RR can be driven by e-cloud with effective density ~ $10^6 cm^{-3}$, being potentially stabilized with the chromaticity > 30.

- The peculiar non-monotonic dependence of the growth rate on the beam population (lower intensity first batch stabilizes the second one independently of its position) could follow from nonlinearity driven by e-cloud trapped inside the gradient magnets as magnetic bottles.

- The HOM hypothesis requires too high Q-values, and thus does not look reasonable.

Many thanks!